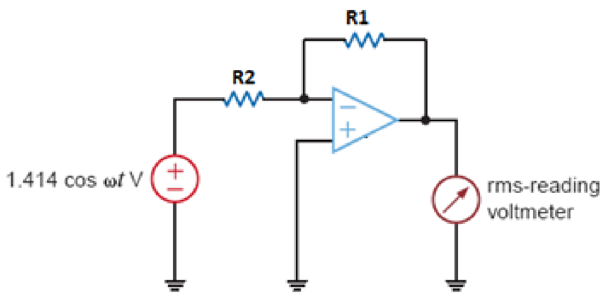
American University of Beirut
Department of Electrical and Computer Engineering
EECE 290 – Analog Signal Processing
Quiz I – Solution

Problem 1(4 pts)

An rms-reading voltmeter is connected to the output of the op-amp shown below. The Op-Amp is operating in its linear mode. Determine the meter reading.



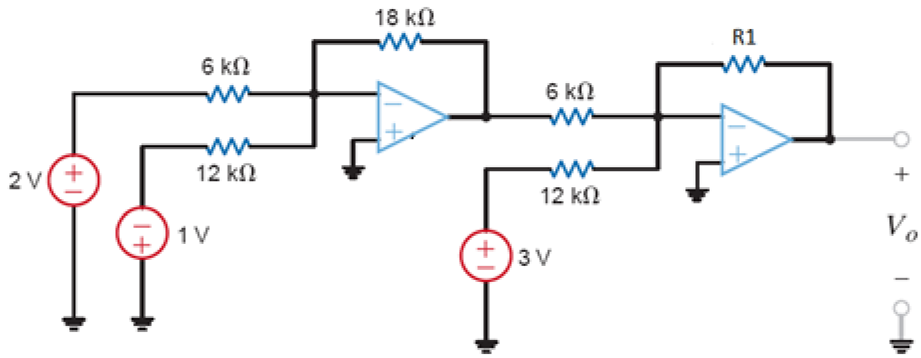
$V_p = 0 = V_n$. Writing Nodal equation at the negative terminal of the op-Amp, we obtain

$$\frac{0-V_{in}}{R_2} + \frac{0-V_o}{R_1} = 0 \text{ and } V_o = -\frac{R_1}{R_2}V_{in}$$

a. If R_1=36 kΩ,R_2=12kΩ, and Vin=1.414<00 Volts, then Vo=3*$\sqrt{2}$<180^0 Volts and rms Value = 3 Volts
b. If R_1=24 kΩ,R_2=6 KΩ, and Vin=1.414<00 Volts, then Vo=4*$\sqrt{2}$<180^0 Volts and rms Value = 4 Volts
c. If R_1=12 kΩ,R_2=2 KΩ, and Vin=1.414<00 Volts, then Vo=6*$\sqrt{2}$<180^0 Volts and rms Value = 6 Volts

Problem 2 (4 pts)

Determine the output voltage V_o of the summing op-amp circuit shown below. Both op-amps are operating in their linear modes.



For the left Op-Amp: $V_p = 0 = V_n$. Writing Nodal equation at the negative terminal of the op-Amp, we obtain

$$\frac{0-2}{6{,}000} + \frac{0+1}{12{,}000} + \frac{0-V_1}{18{,}000} = 0$$

Solve, we obtain V_1= - 4.5 Volts.

For the write Op-Amp, $V_p = 0 = V_n$. Writing Nodal equation at the negative terminal of the op-Amp, we obtain

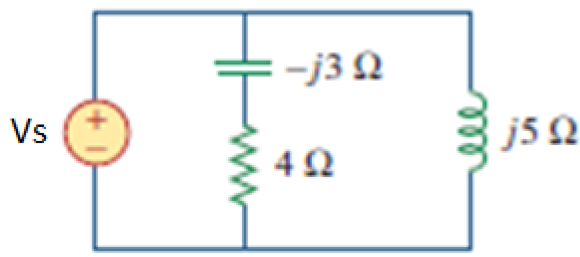
$$\frac{0+4.5}{6{,}000} + \frac{0-3}{12{,}000} + \frac{0-V_o}{R_1} = 0$$

Solve, we obtain $V_0 = 0.5 * 10^{-3} * R_1$

a. For R1 = 36 KΩ, Vo = 18 Volts
b. For R1 = 18 KΩ, Vo = 9 Volts
c. For R1 = 9 KΩ, Vo = 4.5 Volts

Problem 3 (4 pts)

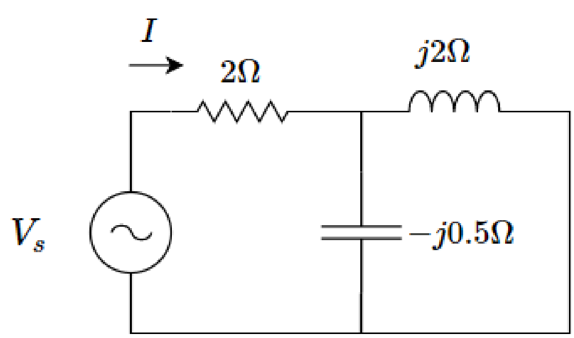
Find the complex power delivered by voltage source if $V_s = x\angle y° \ V$



a. If $V_s = 8\angle - 20° \ V$, then S=$11.456\angle 26.58° \ VA$
b. If $V_s = 16\angle - 20° \ V$, then S=$45.82\angle 26.58° \ VA$
c. If $V_s = 24\angle - 20° \ V$, then S=$103.1\angle 26.58° \ VA$

Problem 4 (4 pts)

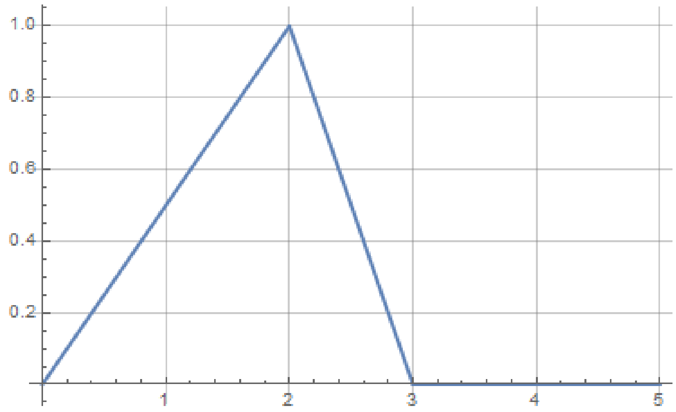
For the given circuit, determine the complex power delivered by the voltage source if $V_s = K \ V \ RMS$ is:



a. If Vs= 10 V rms, then S=(45-j15) VA
b. If Vs=20 V rms, S=(180-j60) VA
c. If Vs=30 V rms, then S=(405-j135) VA

Problem 5 (4 pts) – Solution to Version A

Find the Laplace Transform of the function shown below



f(t) = (0.5t)[u(t)-u(t-2)] + (-t+3)[u(t-2)-u(t-3)]

$$\frac{df(t)}{dt} = \frac{1}{2}[u(t) - u(t-2)] + (0.5t)[\delta(t) - \delta(t-2)] - [u(t-2) - u(t-3)] + (-t+3)[\delta(t-2) - \delta(t-3)]$$

$$\frac{df(t)}{dt} = \frac{1}{2}[u(t) - u(t-2)] - \delta(t-2) - [u(t-2) - u(t-3)] + \delta(t-2)$$

$$\frac{df(t)}{dt} = \frac{1}{2}u(t) - \frac{3}{2}u(t-2) + u(t-3)$$

The Laplace Transform of the derivative of f(t) is :

sF(s)= $\frac{1}{2s} - \frac{3}{2s}e^{-2s} + \frac{1}{s}e^{-2s}$

Therefore,

F(s)= $\frac{1}{2s^2} - \frac{3e^{-3s}}{2s^2} + \frac{e^{-3s}}{s^2}$

Problem 6 (4 pts)

The Laplace transform function for the output voltage of a network is expressed in the following form:

$$Vo(s) = \frac{12(s+2)}{s(s+1)(s+3)(s+4)}$$

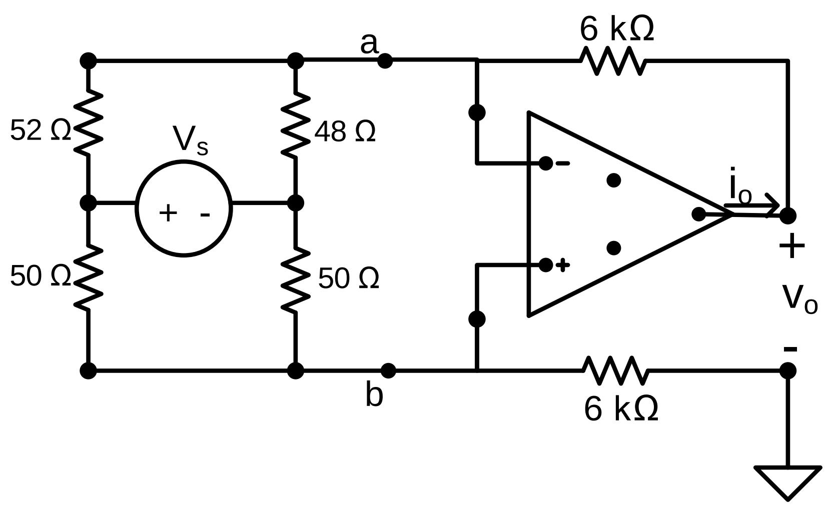
Determine the final value of this voltage that is, $v_o(t)$ as t → ∞.

The final value theorem sates that the limit of $v_o(t)$ as t → ∞ is the limit of $sV_0(s)$ as s goes to zero provided that sVo(s) has no poles either in the rhp nor on the jw axis

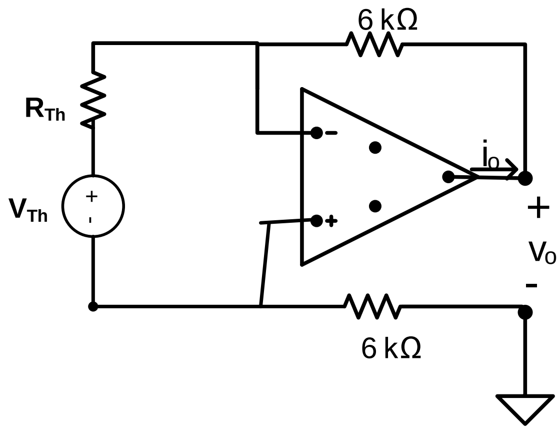
As $sV_0(s)$ satisfies the above criterion, than the limit $v_o(t)$ as t → ∞ is 2 V.

Problem 7 (10 pts)

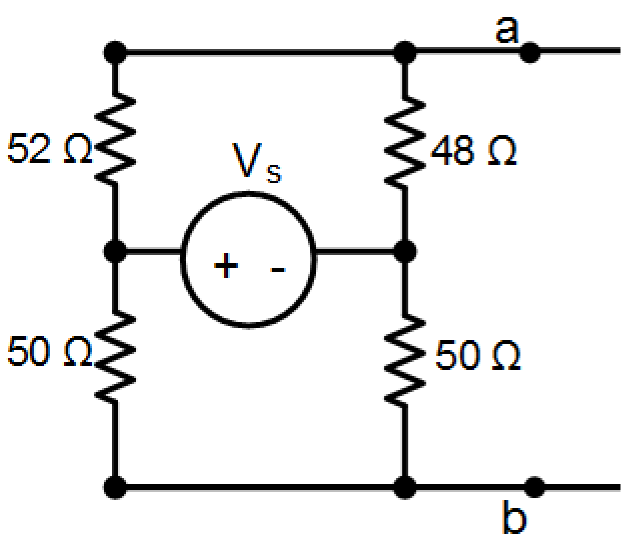
Assume that the Op-Amp operates in the linear region. V_s = ---- V



1. Replace the circuit left of the nodes a-b with a Thevenin equivalent circuit. Draw the new circuit completely. (2 pts)



2. Find the values of V_{Th} and R_{Th}. (4 pts)



If you ground the negative terminal of the voltage, and apply nodal equations at node a, we obtain

$$\frac{V_a - V_s}{52} + \frac{V_a}{48} = 0$$

The above implies,

$$V_a = \frac{48}{100} V_s$$

Applying nodal equations at node b, we obtain

$$\frac{V_b - V_s}{50} + \frac{V_b}{50} = 0$$

The above implies,

$$V_b = \frac{50}{100} V_s$$

$$V_{Th} = V_a - V_b = \frac{48-50}{100} V_s = -0.2 V_s \text{ Volts}$$

And

R_{Th}=48//52+(50//50)= 49.96 Ω

3. Find the value of i_o. (2 pts)

$$V_p = 0 = V_n$$

The current entering R_{Th} is the current i_0 and found to be

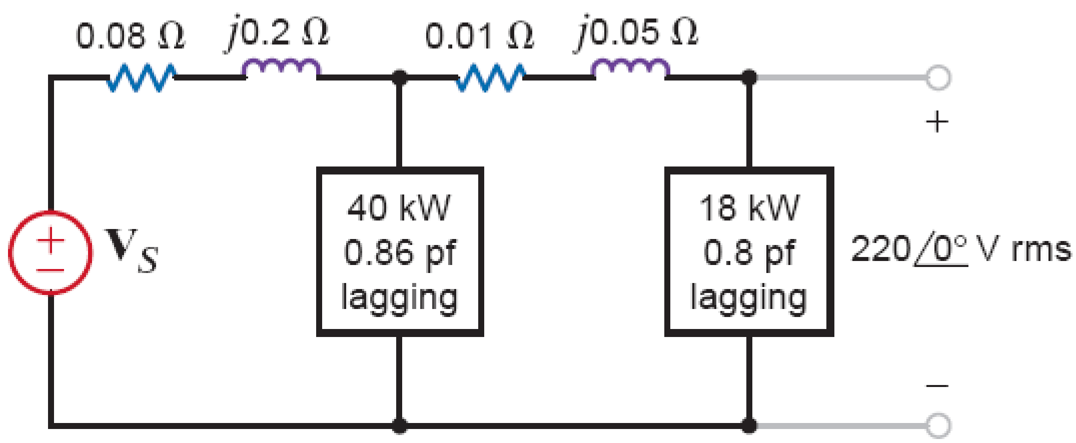
Applying nodal equation at the negative terminal of the op-Amp, we obtain

$$i_0 \frac{0 - V_{Th}}{R_{Th}} = \frac{-0.2 V_s}{49.96}$$

4. Find the value of v_o. (2 pts)

$$i_0 = \frac{V_o}{6 * 10^3} = \frac{-0.2 V_s}{(49.96)(6 * 10^3)}$$

Problem 8 (8 pts)
Given the network below:



1. Find the current in the 18Kw load. (2 pts)

   $I_2 = \frac{P_2}{V_2 P_{f2}} = 102.3 < -36.86^0$ A (rms)

   The above equation is obtained based on the following facts:

   $$p_f = \frac{P}{|S|} = \frac{P}{|V * I^*|} = \frac{P}{|V||I^*|} \xrightarrow{yields} |I| = |I^*| = \frac{P}{|V|P_f} = \frac{18000}{220 * 0.8}$$
   $$= 102.27$$

   The angle of the current lags the angle of the voltage by an angle whose cosine is 0.8. Therefore the angle is 36.86^0

2. Find the current in the 40 KW Load. (2 pts)
   Let V1 be the voltage across the 40 KW load. Using KVL in the right loop, we obtain

   $V_1 = (0.01 + j0.05)I_2 + V_2 = 223.91 < 0.8^0$ Volts (rms)

   Similar to the above,

   $I_1 = 207.7 < -30.68^0$ A (rms)

3. Compute the input source voltage. (2 pts)

   Using KCL, the current Is flowing out of the source Vs is given by:

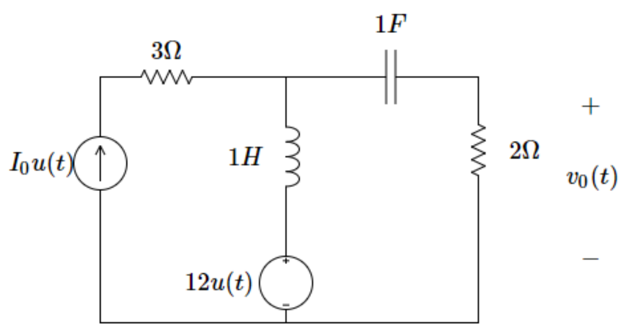
   $I_s = I_1 + I_2 = 309.6 < -32.7^0$ Volts (rms)

4. Compute the input power factor. Is it leading or Lagging? (2 pts)

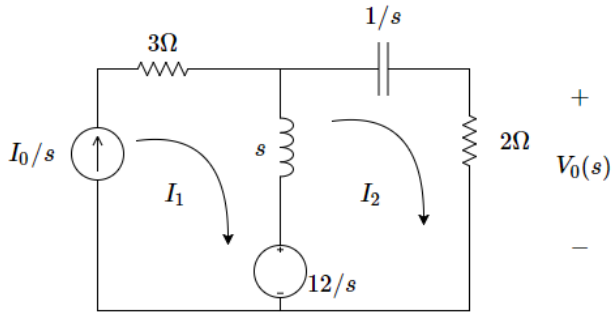
Using KVL in the left loop, we obtain
$V_a = (0.08 + j0.2)I_s + V_1 = 281 < 8.62^0$ Volts (rms)

$$P_f = \cos(\theta_{Vs} - \theta_{Is}) = \cos(8.62 + 32.7) = 0.751\ lagging$$

<u>Problem 9 (8 pts)</u>
Consider the circuit shown below.



Laplace transform, for zero Initial conditions, was used to convert the circuit to s-domain. The following is obtained



a. Write mesh equation for the second mesh I_2(s). Do not simplify. (2 pts)

$$-\frac{12}{s} + s\left(I_2(s) - \frac{I_0}{s}\right) + I_2(s)\left(\frac{1}{s} + 2\right) = 0$$

b. Write I2(s) in fractional format. (2 pts)

$$I_2(s) = \frac{sI_0 + 12}{s^2 + 2s + 1}$$

c. For I_0=1A, Determine v_0(t). (4 pts)

$$V_0(s) = 2I_2(s) = \frac{2I_0 s + 24}{s^2 + 2s + 1} = 2I_0\frac{s}{(s+1)^2} + 24\frac{1}{(s+1)^2}$$

$$\frac{1}{(s+1)^2} \xrightarrow{yields} te^{-t}u(t)$$

$$\frac{s}{(s+1)^2} \xrightarrow{yields} \frac{d}{dt}\left(te^{-t}u(t)\right) = (e^{-t} - te^{-1})u(t)$$

Therefore,

$$v_0(t) = 2I_0(e^{-t} - te^{-1})u(t) + 24te^{-t}u(t) \text{ Volts}$$